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Spatial correlation of tensile perpendicular to grain properties in Norway spruce timber

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Abstract Tensile strength perpendicular to grain constitutes one of the most vulnerable properties of timber. Due to versatile influencing parameters this property exhibits a high amount of uncertainty. Thus, progress in modeling, in particular by considering stochastics, is seen as worthwhile. This increases the reliability estimates of timber constructions but also their economic efficiency. Test data of tensile properties determined on consecutive board segments of Norway spruce are analyzed. The data consists of four subgroups, classified in regard to segment length and radial position within the log. The correlation in longitudinal direction of perpendicular to grain tensile strength and elastic modulus as well as of density is examined. This is done depending on the radial position of structural timber within the log. A second-order hierarchical model together with equicorrelation is used. The results outline the applicability of the model and allow the quantification of equicorrelation coefficients of all three properties. The outcome provides a valuable and necessary input for state-of-the-art mechanics-stochastic modeling of the resistance perpendicular to grain tensile strength and elastic modulus of unjointed and jointed structural timber, but in particular of products available in large dimensions, like glued and cross-laminated timber. Additionally, the spatial correlation of density is discussed which is seen as worthwhile for the estimation of

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group action of fasteners. The necessity to differentiate between the variability within and between segments of structural timber is clearly demonstrated.

Introduction

General

Timber is a natural, sustainable raw material with outstanding properties which offers its use in many fields of application. The material structure optimizes timber for stresses in fiber direction, whereas it is weak in direction perpendicular to grain. In particular, the perpendicular to grain tensile strength exhibits only a minor percentage of the resistance in fiber direction (e.g., Norway spruce clear wood, according to Wagenführ (2006): $f_{t,0,\mathrm{mean}} \approx 90$ N/mm^2, $f_{t,90,\mathrm{mean}} \approx 2.7$ N/mm^2; timber, own experiences: $f_{t,0,\mathrm{mean}} \approx 30$ N/mm^2, $f_{t,90,\mathrm{mean}} \approx 1.0$ N/mm^2). In addition, ultimate failure in tension perpendicular to grain in structural timber elements and products occurs suddenly and brittle. It is obvious that these stresses have to be avoided in timber constructions whenever it is possible. Nevertheless, in some circumstances these stresses are implied ordinary, even in the worst combination, in interaction with shear; for example in tapered, curved and pitched cambered beams, notched beams and openings as well as cross connections. Even in elements which are primarily stressed in tension parallel to grain or in bending the influence of perpendicular to grain tensile stresses on the resistance, due to local (e.g., knots and knot clusters) and global grain deviation, cannot be avoided (Madsen 1992). Thus, enhancing the predictability of the resistance in tension perpendicular to grain and reducing its inherent uncertainty is, of course, worthwhile.

Contrary to the engineering practice, which does not differentiate between the resistance in radial and in tangential fiber direction, Blaß and Schmid (1999, 2001), Dill-Langer (2004), and others demonstrated clearly the influence of the annual ring pattern on the distribution of tensile stresses. Shear coupling due to the transformation of stresses and strains from Cartesian to the more realistic cylindrical coordinate system shows stress concentrations which occur primary in radial direction and in specimens taken near the pith, even in cases where the load is applied uniformly. This aspect requires consideration when size effects are discussed. In fact, tension perpendicular to grain strength underlies a distinctive size effect which is, beside the disregard of its boundary conditions and assumptions, often modeled by means of Weibull's brittle failure theory (Weibull 1939a, b).

Size effects and Weibull's model constraints

Weibull (1939a, b) assumed independent and identically distributed (iid) random flaws of infinitesimal size. He modeled the material as continuum and applied the linear elastic theory. Of course, flaws in timber, like knots or knot clusters, are distributed in some geometrically regular pattern. Their dimensions are even at the same dimensional scale as the timber specimen. Despite this, the mathematical model behind Weibull's theory was used successfully for explaining the differences

in resistances even of arbitrary stressed volumes of tapered, curved and pitch-tapered beams as well as cross connections (e.g., Barrett et al. 1975). Weibull's further assumption of an ideal brittle material behavior is often not confirmed by test experience. Mistler (1979), Dill-Langer (2004), Stuefer (2011) and others observed partial failures and further load increase in a remarkable number of tests (e.g., 60 % and more; Dill-Langer 2004). In his proposed stepwise failure scenario, Dill-Langer (2004) outlines partial failure and crack formation before a sudden brittle failure, even up to two-third of specimen's width. Mistler (1979) stated that an increase in volume must not necessarily coincide with a decrease in strength (e.g., increasing $f_{t,90,mean}$ with increasing width in GLT; Blaß and Schmid 1999) and in particular with an ideal serial system action. Thus, a more realistic modeling can be achieved by differentiation between parallel and serial system action, considering the principal ideas of Weibull (1939a, b) and the fiber bundle model of Daniels (1945) (see, e.g., Zweben and Rosen 1970; Smith 1982; Harlow et al. 1983; Phoenix et al. 1997; Ibnadeljalil and Curtin 1997; Sutherland et al. 1999 a.o.). Overall, tensile perpendicular to grain strength is known to be vulnerable to many more parameters like checks, splits, pitch pockets and pith (Blaß and Schmid 1999, 2001; Dill-Langer 2004 a.o.), but also to moisture-induced stresses caused by climate variations (e.g., Toratti 1992).

Uncertainty in tension perpendicular to grain properties

A high variability in tensile perpendicular to grain strength can be observed even by testing graded timber under constant climatic conditions. This remarkable uncertainty, inherent in this property, necessitates its consideration in design procedures. In recent times, engineers and researchers have tried to circumvent this challenge by applying reinforcements. Hereby, the need to verify tension perpendicular to grain resistance is in principle disabled. Recently, screw rods were proven to not really reinforce but rather weaken timber structures by provoking cracks as a consequence of inevitable climate variations (Wallner 2012). Thus, there is evidence to further increase knowledge with focus on more accurate predictions of tensile perpendicular to grain resistance. This is in fact seen as worthwhile for improving decisions whether or not reinforcements are required, and perhaps also for designing them more accurately.

Hierarchical models and equicorrelation

Within this paper, the principle ideas of Weibull (1939a, b), Daniels (1945) and of subsequent papers in the field of stochastic material modeling are seen as motivation to treat perpendicular to grain tensile properties as a function of serial and parallel system effects. These system effects follow from (common) action of representative elements. For example, a serial system action occurs between the lamellas of an edgewise-stressed glulam beam, whereas an increase in width can be assigned to a parallel system action (see also Mistler 1979). Comparable thoughts have already been made by Bohannan (1966) who focused on size effects of timber in bending, and recently and more generally by Brandner (2012).

In fact, the magnitude of stochastic system action is directly a function of the variability of the analyzed property. Thus, quantification of stochastic serial and parallel system action necessitates at first to allocate the overall variability observed in full-scale tests to variability within and between structural timber elements. Therefore, the simplest case of a second-order hierarchical model, which has already been proven to be successful in modeling timber strength properties, is applied (see, e.g., Riberholt and Madsen 1979; Colling 1990; Williamson 1992, 1994; Källsner et al. 1997; Isaksson 1999; Ditlevsen and Källsner 2005; Köhler 2007). The model follows

$$Z_{ij} = Y_j + X_{ij},\ \text{with}\ i,\quad j = 1, \ldots, m \tag{1}$$

with Y_j as mean value of the analyzed property of the jth element and X_{ij} as iid deviation from Y_j by subelement i as part of element j, with expectation $\mu_X = \mathrm{E}[X_{ij}] = 0$, variance $\mathrm{Var}[X_{ij}] = \sigma_X^2$, covariance $\mathrm{CoVar}[Y_j + X_{ij}, Y_j + X_{kj}] = \mathrm{Var}[Y_j] = \sigma_Y^2$, $\mathrm{E}[Z_{ij}] = \mathrm{E}[Y_j] = \mu_Y$ and $\mathrm{Var}[Z_{ij}] = \sigma_X^2 + \sigma_Y^2$. Thus, a so-called equicorrelation coefficient ρ_{equi} can be defined as (see, e.g., Källsner et al. 1997):

$$\rho_{\mathrm{equi}} = \frac{\sigma_Y^2}{\sigma_X^2 + \sigma_Y^2}. \tag{2}$$

In probability theory, equicorrelation constitutes the simple stochastic case where all coefficients in the correlation matrix $\boldsymbol{\rho}$ of size $m \times m$ of m variables in a set are equal, with $\rho_{ij} = \rho_{\mathrm{equi}}$, for $i, j = 1, \ldots, m$ and $i \neq j$.

In fact, this hierarchical model can be directly inferred by the hierarchical material structure of wood and timber apparent on several scales (see, e.g., Brandner 2012).

The allocation of total variability in variability between and within timber elements can be done directly by means of the equicorrelation coefficient. In the following, the focus is on the examination of the correlation between segments observable within the same element. This information constitutes a valuable input for more realistic modeling of tensile perpendicular to grain properties of elements with and without joints in dependency of the element length and the number of joints. This is in particular seen as relevant for glulam and other structural timber products.

Thus, the aims of this paper are (1) to quantify the longitudinal pairwise correlation of tensile perpendicular to grain elastic modulus and strength as well as density, (2) to prove whether the assumption of equicorrelation for these properties is correct and (3) to analyze the influence of the radial position within the log, as parameter for the annual ring pattern, on the correlation.

Materials and methods

The data set analyzed is extracted from Stuefer (2011).

Test material

The tested material of grade L25 according to EN 14081-4:2009 with the additional requirement on the density at 12 % moisture content, $\rho_{12} > 390\ \mathrm{kg/m^3}$,

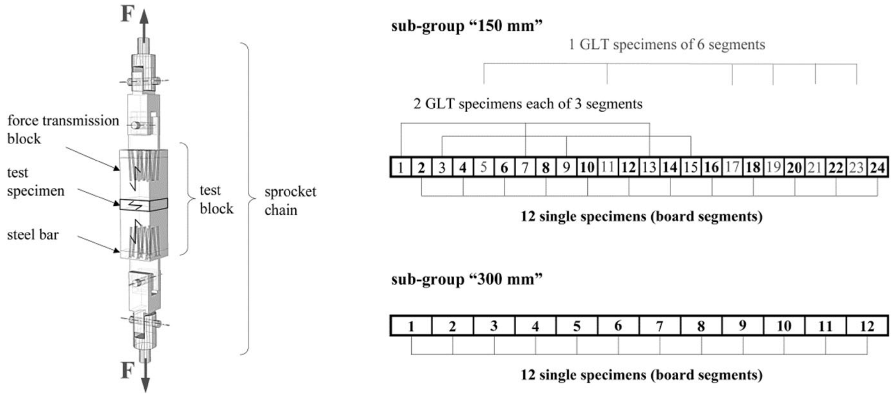

Fig. 1 Test configuration (*left*; adapted from Stuefer 2011) and classification schema for subgroups "150 mm" and "300 mm" according to the segment length (*right*)

was taken from boards of Central European Norway spruce (*Picea abies* (L.) Karst). The dimensions at delivery were $w/t/l = 210$ mm/46 mm/4,030 mm. After planning, with the aim that the width is centered to the pith, the nominal dimension of the cross section was $w/t = 150$ mm/40 mm. In total, 24 out of 110 boards were further processed and classified in two equally sized groups according to their radial position within the stem (RP): group M was defined by 20 mm $<$ RP $<$ 80 mm and group S by RP $>$ 80 mm. Apparent pith was excluded. Each group was again divided into two equally sized subgroups for further trimming in 24 and 12 segments of nominal length of 150 and 300 mm, respectively, with an average center distance of 165 and 330 mm. Segments with length 150 mm were either used directly for testing or for composing glulam specimens with three (two specimens for each board) and six (one specimen for each board) segments each, see Fig. 1. All material was conditioned and stored at 20 °C and 65 % relative humidity in a climate chamber before trimming and planning and until testing.

Test configuration and applied test methods

Density ρ_{12} and dynamic elastic modulus $E_{0,12,\mathrm{dyn,EF}}$ based on eigenfrequency were determined on all boards before trimming. Furthermore, all surfaces were scanned for determination of the dimension and position of flaws, e.g., knots, knot clusters, resin pockets and checks. After trimming, all cross sections were scanned for determination of the radial position within the log (RP) and the annual ring width (ARW). Here, RP is defined as the radial distance between the geometrical center of the board and the pith. In addition, the density of every segment was determined.

Tensile tests perpendicular to grain were performed according to EN 408:2009 on so-called test blocks. They consisted of a flatwise-placed specimen in the center (in compliance with its orientation in glulam), shielded by two force transmission blocks glued on in cross-grained direction (see, e.g., Ehlbeck and Kürth 1994;

Fig. 1). The material of the force transmission blocks was again Norway spruce, with an average density of $\rho_{12} = 450\ \text{kg/m}^3$ and an average dynamic elastic modulus of $E_{0,12,\text{dyn,EF,mean}} = 13{,}000\ \text{N/mm}^2$. To ensure centered loading, the test setup was designed as sprocket chain. Force transmission from the steel bar hinged to the testing device into the test block was obtained by 12 and 24 full-threaded self-tapping screws (Schmid "Star Drive" 8 × 160 mm), for 150- and 300-mm-long specimen, respectively, inserted inclined by 10° (inner screws) and 5° (outer screws) with respect to the longitudinal axis. The stiffness of steel bars was designed to prevent the deflection to exceed 0.1 mm at the utmost corners of the 300-mm-long elements at the maximum load of the testing device of 275 kN.

Local deformations were measured on all four sides of the test specimen by means of DD1 transducers of HBM and with a test frequency of 5 Hz and a base of 35 mm. Further details are given in Stuefer (2011).

Statistical inference

Statistical analysis and inference was mainly performed in R (R Core Team 2012). After calculation of basic statistics, the focus was on testing the pairwise correlation between segments of all four subgroups and all three properties, tensile elastic modulus and strength perpendicular to grain as well as density. An approximate statistic for testing the hypothesis H_0: $\rho_{\text{ij}} = \rho = \rho_{\text{equi}}$ versus H_1: $\rho_{\text{ij}} \neq \rho_{\text{equi}}$, for $i, j = 1, 2, \ldots, m$ and $i \neq j$, with m as the number of random variables and n as the number of realizations per variable, with the general requirement of $n \gg m$, is given as (see Lawley 1963)

$$\hat{t}_n = \frac{n-1}{(1-\bar{\hat{r}})^2} \cdot \left[\sum\sum_{i<j} (\hat{r}_{ij} - \bar{\hat{r}})^2 - \hat{\omega} \cdot \sum_{j=1}^{m} (\bar{\hat{r}}_j - \bar{\hat{r}})^2 \right], \tag{3}$$

with

$$\bar{\hat{r}}_j = \frac{1}{m-1} \cdot \sum_{i \neq j}^{m} \hat{r}_{ij}, \quad \bar{\hat{r}} = \frac{2}{m \cdot (m-1)} \cdot \sum\sum_{i<j} \hat{r}_{ij} \text{ and } \hat{\omega} = \frac{(m-1)^2 \cdot \bar{\hat{r}} \cdot (2 - \bar{\hat{r}})}{\left[m - (m-2) \cdot (1 - \bar{\hat{r}})^2 \right]} \tag{4}$$

which follows asymptotically a χ^2 distribution

$$\hat{t}_n \overset{asym.}{\sim} \chi^2_{f,\alpha}, \text{ with } \quad f = \frac{1}{2} \cdot (m+1) \cdot (m-2) \tag{5}$$

degrees of freedom. Hereby, it is assumed that the m variables follow a multivariate normal distribution. Estimates $\hat{r}_{ij}$ correspond to the Pearson's sample correlation coefficients. The null hypothesis has to be rejected whenever $\hat{t}_n$ exceeds the $(1 - \alpha)$-quantile of the χ^2 distribution. The best estimator for the equicorrelation coefficient ρ_{equi} is represented by the weighted averaged correlation coefficient $\bar{r}$ gained from correlation matrix $\hat{R} = \hat{r}_{ij}$ of test results (n realizations of m variables), with dimension $m \times m$.

Results and discussion

General characterization of test material and main statistics

At first, data sets of segments with end grain checks as well as extreme outliers were excluded from further data processing. Unexpectedly, most of the segments of one board in subgroup "300_M" included pith. Thus, all data sets of this board had to be rejected too. In this respect, a comparison by Stuefer (2011) of tension perpendicular to grain strength between unchecked and checked segments as well as between segments with and without pith, respectively, indicated a reduction in median of 50 and 45 %. The quantified influence of pith is thus in line with Blaß and Schmid (2001). These first results indicate the necessity to exclude pith and to prevent timber from excessive cracking due to drying processes but in particular during the lifetime of constructions. Otherwise, the regulated resistance has to be on a very conservative basis.

Table 1 shows averages of basic material properties ARW, RP, the knot area ratio (KAR), as a relative measure for the occurrence of knots and knot clusters within 150 mm segment length, and the moisture content u at time of testing in tension. As expected, ARW and KAR decrease with increasing RP.

The presence of knots was found to have a minor influence on the tensile perpendicular to grain properties. Stuefer (2011) reports a 5 % lower median in specimen with knots. A significant difference between the medians of specimen with and without knots could not be confirmed (Mann–Whitney U Test, $p < 0.05$). The same magnitude of influence was found for resin pockets which were present on fractured surfaces in 28 % of all specimens. Thus, segments with knots and/or resin pockets were left in the analysis. The main statistics [minimum, maximum, averages, medians and coefficients of variation (CoV)] of density, tension perpendicular to grain strength and elastic modulus of the segments as well as the dynamical elastic modulus of the boards tested in each subgroup are given in Table 2. Hereby, average values of strength and elastic modulus are found to increase with increasing RP. This can be partly attributed to the differences between juvenile and adult wood. Nevertheless, this observation cannot be confirmed for density due to the fact that this property was used directly for material grading. Nevertheless, $E_{\mathrm{dyn,EF,12}}$ of subgroups "S" indicates that this material may be attributed to a strength class higher than L25.

By comparing strength values of subgroups "150" and "300", a decrease with increasing area stressed in tension perpendicular to grain can be observed. Overall,

Table 1 Average values (min–max) of basic material properties

Subgroup (–)	ARW (mm)	RP (mm)	KAR (–)	u (%)
150_M	2.7	44 (28–83)	0.14	12.2
150_S	2.4	119 (75–161)	0.09	12.7
300_M	3.6	41 (23–61)	0.19	12.2
300_S	2.2	118 (75–167)	0.10	12.9

Table 2 Main statistics of test data for each subgroup

Subgroup (–)	Statistic	$\rho_{12,ij}$ (kg/m^3)	$E_{0,12,dyn,EF,j}$ (N/mm^2)	$f_{t,90,ij}$ (N/mm^2)	$E_{t,90,ij}$ (N/mm^2)
150_M	No.	70	6	70	70
	Min	412	10,450	1.23	670
	Average	468	11,540	2.47	860
	Median	473	11,600	2.36	860
	Max	520	12,290	4.12	1,170
	CoV (%)	5.5	5.8	26.7	14.5
150_S	No.	71	6	71	71
	Min	407	10,000	1.07	640
	Average	441	13,300	2.85	900
	Median	439	13,650	2.99	870
	Max	491	15,870	4.83	1,420
	CoV (%)	5.0	15.0	29.0	14.8
300_M	No.	56	5	56	56
	Min	382	9,590	1.14	610
	Average	427	11,350	1.81	800
	Median	430	10,750	1.83	800
	Max	477	13,540	2.72	1,070
	CoV (%)	5.9	13.8	20.1	14.2
300_S	No.	69	6	69	69
	Min	406	11,600	1.05	780
	Average	457	14,640	2.56	930
	Median	456	14,700	2.59	940
	Max	518	17,380	4.31	1,070
	CoV (%)	6.2	14.2	25.6	8.1

the average strengths and their variations are in line with results by Blaß and Schmid (2001), in particular when the annual ring pattern, and thus, primary radial stressing is considered. In reference to them, average values of 2.55 and 1.80 N/mm^2 were found for radial and tangential direction, respectively. Thus, the differences in $f_{t,90,mean}$ between subgroups of “M” and “S” can be attributed to changes in the annual ring pattern, from combined radial and tangential in “M” to primary radial in “S”. In contrast, values of elastic modulus in tension perpendicular to grain ($E_{t,90}$) are unexpectedly high. Although a remarkable difference between $E_{t,90}$ in radial and tangential direction is known (e.g., according to Blaß and Schmid 1999: $E_{t,90,mean} = 726$ and 164 N/mm^2 for radial and tangential direction, respectively), the high values, in particular in subgroup “M”, are not seen as representative, even if the main part of segments is stressed radially. The reason for these high values in subgroup “M” is attributed to the placement of DD1 transducers on all four sides of the specimens. As values of $E_{t,90}$ were simply calculated as averages no balancing according to the stress distribution over the cross section was made. Nevertheless, as the main focus of this paper is on the

Table 3 Statistics of the coefficients of variation (CoV) gained from properties observed on segments of the same board: average values (min–max)

Subgroup (–)	CoV[$\rho_{12,\mathrm{ij|j}}$] (%)	CoV[$f_{\mathrm{t,90,ij|j}}$] (%)	CoV[$E_{\mathrm{t,90,ij|j}}$] (%)
150_M	2.4 (1.7–3.3)	20.0 (13.4–25.5)	8.7 (4.3–16.5)
150_S	1.6 (0.9–3.1)	22.2 (11.4–33.4)	8.9 (4.9–16.9)
300_M	2.7 (1.9–4.8)	18.0 (13.3–23.2)	10.8 (7.2–13.8)
300_S	1.6 (0.9–2.0)	19.0 (10.6–33.6)	5.8 (3.3–7.6)

pairwise correlation, the influence of this absolute bias is judged as negligible as the method applied for determination was kept constant and the variation of annual ring pattern in each subsample was under control.

Table 3 gives average values, minima and maxima gained from estimates of the coefficients of variation $CoV\left[Z_{ij|j}\right] = \sqrt{Var\left[Z_{ij|j}\right]}\Big/E\left[Z_{ij|j}\right]$ from properties observed on segments of the same board. Thus, estimates of CoV[$Z_{\mathrm{ij|j}}$] are calculated by relating the realized averages of property $Z_{\mathrm{ij|j}}$, observed on i segments of the jth board, to the sample standard deviation, calculated from the same realizations. In comparison with Table 2, it is obvious that the variabilities within boards are smaller. This circumstance is in particular evident for density, whereas for strength, nearly equal values are found. This already indicates that the pairwise correlation within the boards is lower for tensile perpendicular to grain strength than for density.

Analysis in regard to longitudinal trends

Before testing the hypothesis of equicorrelation, every data set per board was analyzed in regard to longitudinal trends. Hereby, gradients of simple linear regression models adapted to these data sets were tested with null hypothesis H_0: Gradient is zero, and the alternative hypothesis H_1: Gradient is not zero. The hypothesis H_0 could not be rejected in most cases ($p < 0.05$). Only for density in group "300", hypothesis H_0 was rejected in half of the boards in both subgroups, "S" and "M". As no common trend was found and for lack of a general applicable, reliable model detrending was not performed.

Tests on equicorrelation

Lawley's test on equicorrelation is based on multivariate normal distributed variables. According to the recommendations of the Joint Committee on Structural Safety (2006), density, tensile perpendicular to grain strength and elastic modulus should be modeled by a normal (ND; CoV[ρ_{12}] = 10 %), a two-parameter Weibull (2pWD; CoV[$f_{\mathrm{t,90}}$] = 25 %) and a two-parameter lognormal distribution (2pLND; CoV[$E_{\mathrm{t,90}}$] = 25 %), respectively. In fact, Brandner (2012) and others found a lognormal distribution to be more representative for density. Although 2pWD is

Table 4 Results of Lawley's test on sample equicorrelations of untransformed (linear) and transformed (log) data: best point estimators, p values and weighted standard error s_e

Subgroup	Statistic	ρ_{12}		$E_{t,90}$		$f_{t,90}$	
		Linear	Log	Linear	Log	Linear	Log
150_all	r_{equi}	0.89	0.89	0.64	0.63	0.50	0.46
	p	0.663	0.737	0.333	0.403	0.137	0.091
	s_e	0.029	0.027	0.104	0.115	0.086	0.097
150_M	r_{equi}	0.86	0.87	0.78	0.76	0.42	0.38
	p	0.281	0.327	0.094	0.042	0.009	0.018
	s_e	0.076	0.073	0.139	0.180	0.120	0.133
150_S	r_{equi}	0.92	0.91	0.57	0.57	0.52	0.52
	p	<0.001	<0.001	0.139	0.180	0.066	0.066
	s_e	0.039	0.041	0.193	0.173	0.109	0.109
300_all	r_{equi}	0.90	0.89	0.74	0.74	0.59	0.55
	p	<0.001	<0.001	0.221	0.124	<0.001	0.001
	s_e	0.036	0.037	0.053	0.071	0.091	0.098
300_M	r_{equi}	0.82	0.83	0.69	0.68	0.16	0.17
	p	0.050	0.056	0.001	0.007	0.010	0.009
	s_e	0.059	0.058	0.128	0.149	0.264	0.257
300_S	r_{equi}	0.96	0.96	0.55	0.56	0.46	0.44
	p	<0.001	<0.001	0.362	0.315	0.001	<0.001
	s_e	0.025	0.027	0.103	0.105	0.140	0.155

used commonly for $f_{t,90}$, for example, Blaß and Schmid (2001) found the 2pLND to be more representative. Comparing the empirical distributions of Stuefer's (2011) data, used in the current analysis, with ND, 2pLND and 2pWD, again the distribution model of Weibull was found to represent the data best. Thus, Lawley's test on equicorrelation can only be used approximately. Of course, for (nearly) lognormal distributed variables, the simple transformation $Y = \ln(X)$, with $X \sim$ 2pLND, and $Y \sim$ ND allows the application of Lawley's test without further constraints. Thus, for comparison, analysis is made on untransformed and transformed values of $\rho_{12,ij}$, $E_{t,90,ij}$ and $f_{t,90,ij}$ (see Table 4). The equicorrelation coefficients $r_{equi} = \bar{\hat{r}}$ are calculated according to Eq. (4). Furthermore, calculation of the (longitudinal) pairwise correlation of the samples is performed by means of Pearson's correlation coefficient $\rho_{pw,P}$ (point estimate and 95 % confidence band), which again base upon normal distributed variables, and by the rank correlation coefficient of Spearman, $\rho_{pw,SP}$, a correlation coefficient free of parameters. The pairwise correlations are calculated by successive enlargement of the distance between the paired segments of the same board of which the data is used, with $r_{pw}(Z_{ij|j}, Z_{(i+k)j|j})$ and lag-distance $k = 0, 1, \ldots, (i-1) = 0, 330, \ldots, 3{,}630$ mm. This lag-distance corresponds to the distance between the geometric centers of the segments in longitudinal direction. The results are visualized in Fig. 2.

First of all, differences between estimated equicorrelations of untransformed and transformed data sets as well as between the courses of Spearman's and Pearson's correlation coefficients are minor and overall negligible. This is true for all series and properties beside the density in subgroup "300". Here, the results confirm the trends found before. Testing on equicorrelation after detrending by means of a simple linear regression model gives estimates of $r_{\mathrm{equi}} = 0.92$ ($p = 0.1079$) and $r_{\mathrm{equi}} = 0.97$ ($p \approx 0$) for "M" and "S", respectively.

Comparability between results, irrespective of variable transformation and calculation method of pairwise correlation, indicates minor influence beside the violation of normal distribution in herein analyzed data.

Comparison of the results shown in Fig. 2 together with Table 4 confirms the general assumption that density within the same board is much more correlated than perpendicular to grain tensile elastic modulus and strength. Thereby, strength values show the lowest correlations within specimens. This is simply explained by the high dependency of strength on local timber properties, in particular on flaws, and thus by the fact that strength represents absolutely a local property. In contrast, the density is an average value of the whole-material volume under investigation. The correlation of elastic moduli is somewhere in between. On the one hand, it is strongly related to local properties, and on the other hand, it constitutes a weighted property of the related material volume. Figure 2 also demonstrates the increasing 95 % confidence band in estimated pairwise correlations and the increasing uncertainty in equicorrelations with increasing variability in the analyzed property. Beside of the results for density in subgroup "300", none of the courses of pairwise correlations indicate qualitatively distinctive longitudinal trends. Despite this, the assumption of equicorrelation is not always confirmed by Lawley's test (see Table 4). In particular, tests in regard to tension strength are often rejected ($p < 0.05$).

Table 4 gives estimates s_{e} of the standard error of observed residual e_{i}, which is defined as difference between pairwise correlation and equicorrelation. It gives a measure of the uncertainty, weighted according to the number of realizations. The contradiction between qualitative and quantitative judgment can be explained by the low number of realizations. In general, it is required that the number of realizations is much larger than the number of variables ($n \gg m$). However, in the analyzed data set, m is equal to 12 for $n \leq 12$ and 6 in cases were groups and subgroups are analyzed. In general, the correlation of $f_{\mathrm{t,90}}$ in subgroup "300_M" is much more heterogeneous than in the rest of the data set, although the variability between the segments is in magnitude comparable with that of the other subgroups.

As expected, comparing the results for density and strength of subgroups "M" with "S", it can be observed that the correlation between segments of boards taken near the pith is lower than in segments of boards taken from the outer region of the log. For $E_{\mathrm{t,90}}$, the opposite is given, even for both subgroups "150" and "300". This result is in fact unexpected and contrary to the general expectation of decreasing correlation with decreasing timber quality, which normally coincides with a reduction in RP. Several previous examinations confirm this expectation for different strength and stiffness properties, e.g., Riberholt and Madsen (1979), Leicester (1985), Taylor (1988), Taylor and Bender (1991), Richburg and Bender (1992), Lam et al. (1994), Williamson (1994) and Brandner et al. (2005).

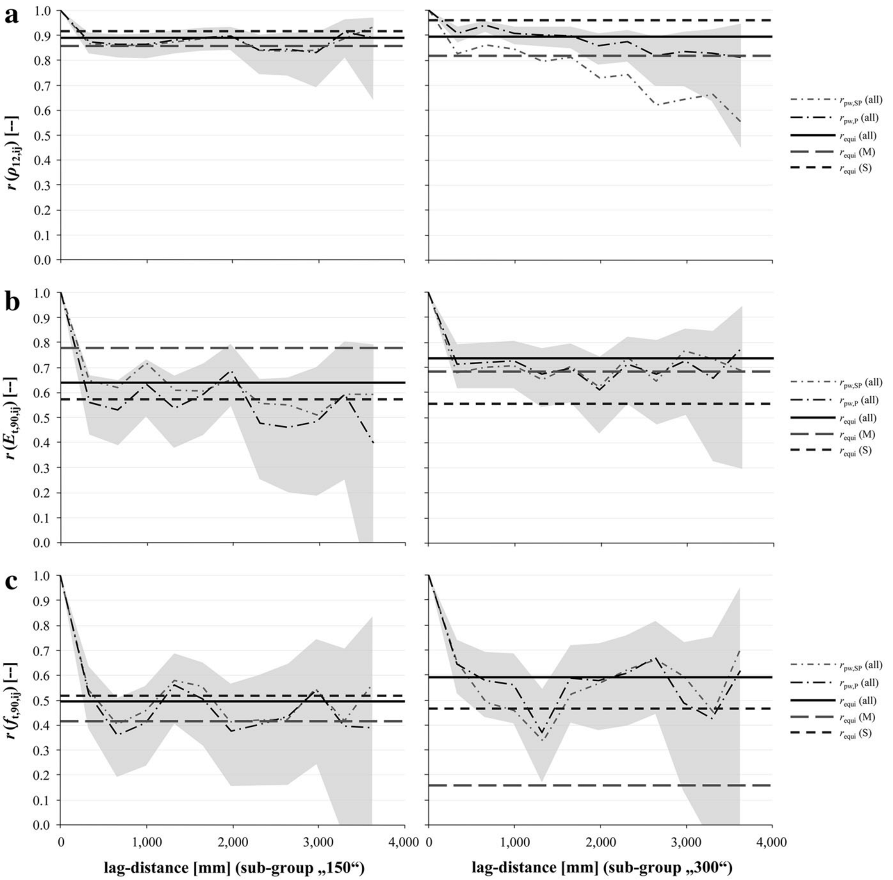

Fig. 2 Pairwise correlations and equicorrelations of samples: **a** density $\rho_{12,\mathrm{ij}}$ **b** elastic modulus $E_{\mathrm{t},90,\mathrm{ij}}$ **c** strength $f_{\mathrm{t},90,\mathrm{ij}}$

In general, it is known that the core of a log is dedicated to juvenile wood which is characterized by a significant change of wood and timber properties and high variability in occurrence and magnitude of flaws. Boards with a higher RP contain more adult wood. This is characterized by more homogeneous wood properties. Beside this radial change in wood properties, also a change in longitudinal direction of the log, from the base to the top, is given (see e.g., Brandner 2012). Thereby, zones (1) free of knots, (2) with green and (3) with dead knots can be distinguished. These zones cause some additional variability in timber properties, in particular in zones which are influenced locally by flaws. Consequently, at a given RP, the occurrence of flaws in longitudinal direction of boards is in general more homogeneous near the pith (low RP-values), whereas it becomes more heterogeneous in specimen with higher RP. Thus, the observation of decreasing equicorrelation in $E_{\mathrm{t},90}$ with increasing RP may be attributed to a raising variability in the occurrence of flaws.

By analyzing the data of subgroups "150" and "300" in regard to possible differences in equicorrelations, no trends are found. It can be concluded that for investigations on the pairwise correlation within and between timbers in regard to perpendicular to grain tensile properties both, 150- and 300-mm-long specimens give comparable results. In cases, where strength or stiffness properties are known experimentally from specimens with comparable dimensions, herein calculated equicorrelations can be used directly for modeling of larger and more complex structures.

To simplify the modeling of structures stressed in tension perpendicular to grain or of the spatial distribution of density in regard to group action of fasteners (hereby density is so far the only indicating timber property used for estimating the withdrawal and embedment capacities), homogenized equicorrelation coefficients are synthesized from Table 4 and following ranges are proposed: $\rho_{\mathrm{equi}}(\rho_{12,\mathrm{ij}}) = (0.80–0.90)$, $\rho_{\mathrm{equi}}(E_{\mathrm{t},90,\mathrm{ij}}) = (0.60–0.70)$ and $\rho_{\mathrm{equi}}(f_{\mathrm{t},90,\mathrm{ij}}) = (0.40–0.50)$. Thereby, higher values of equicorrelation correspond to more homogeneous material (i.e., higher strength class), e.g., due to grading, and/or for material taken from the adult wood zone. The difference in equicorrelation due to changes in RP on density, strength and elastic modulus was found to be approximately 0.10.

The proposed range of $\rho_{\mathrm{equi}}(f_{\mathrm{t},90,\mathrm{ij}})$ is more or less comparable to the results of local bending, tension and compression strength published by Riberholt and Madsen (1979), Leicester (1985) and Stich (1998). Showalter (1986), Madsen (1989), Källsner et al. (1997), Ditlevsen and Källsner (1998) and Isaksson (1999) found slightly higher values, mostly in the range of 0.50–0.60, with a maximum of 0.72. The higher equicorrelations can be partly explained by a lower variation in the analyzed characteristic, e.g., bending strength, as consequence of a more homogeneous material.

Results in regard to equicorrelation of elastic moduli are scarce. Whereas Hoffmeyer (1987) report on higher values (0.81–0.85), based on data of Brandner et al. (2005), lower coefficients in the range of 0.36–0.41 are found. Nevertheless, taking into account results from authors who performed general autocorrelation analysis, a more conservative range of $\rho_{\mathrm{equi}}(E_{\mathrm{t},90,\mathrm{ij}}) = (0.50–0.60)$ was found in Brandner (2012). In contrast, for elastic modulus of tension and compression parallel to grain, Colling (1990) considered relatively high equicorrelations, e.g., $\rho_{\mathrm{equi}}(E_{\mathrm{t},0,\mathrm{ij}}) \approx 0.80$, which are much higher than found on average.

Concerning $\rho_{\mathrm{equi}}(\rho_{12,\mathrm{ij}})$, so far no literature source is known which quantifies this property in regard to its spatial correlation. Nevertheless, e.g., Colling (1990) assumed for simplicity a constant value of density within a board, with $\rho_{\mathrm{equi}}(\rho_{12,\mathrm{ij}}) = 1.00$. Focusing on the spatial distribution of density in regard to the group action of fasteners, it is also required to consider the discrete distribution of flaws, i.e., knots. Due to their density being two to three times higher than in the surrounding wood, an increase, e.g., in withdrawal strength of comparable magnitude is known from tests.

Conclusion

After a brief introduction to the latest findings in regard to tension perpendicular to grain properties, often performed violations of Weibull's constraints and assumptions made to derive his brittle failure theory are outlined. As a consequence and under consideration of current stochastic material modeling, the necessity to differentiate between serial and parallel system action and effects by combining the principle ideas of both theories, Weibull (1939a, b) and Daniels (1945), is demonstrated.

Due to the apparent hierarchical structure of wood and timber over various scales, a second-order hierarchical model with assumed equicorrelation is defined and further used for analyzing sequential data of perpendicular to grain tensile strength and elastic modulus as well as density. Based on a data set of Stuefer (2011), consisting in total of four subgroups, equicorrelations are qualitatively verified. Nevertheless, statistical inference by means of Lawley's test rejected partly the assumption of equicorrelation ($p < 0.05$), in particular for strength values. Based on qualitative results and the fact that transformation from assumed lognormal to normal domain as well as comparison of Spearman's and Pearson's pairwise correlations give no significant differences (beside density in subgroup "300" which suffers from trends in data), further discussion retained on equicorrelation.

Ranges for the parameter equicorrelation of all three timber properties are defined. As in general, a higher equicorrelation can be expected in a more homogeneous material, e.g., secured by successful grading and/or by taking timber from the adult wood zone, associated with higher strength class timber. The comparison of proposed values with literature confirms the ranges for strength and elastic modulus associated with other types of stresses. The equicorrelations are seen as valuable and necessary input for the state-of-the-art stochastic-mechanic material and product modeling in regard to perpendicular to grain tensile resistances, as well as density, here for modeling of group actions between fasteners.

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